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Quarkonium mass spectra from the temperature dependent Bethe-Salpeter equation with logarithmic and Coulomb plus square-root kernels

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Abstract. The study of meson formation in quark-gluon plasma using a temperature-dependent Bethe-Salpeter formalism presented earlier, is now extended to a logarithmic and a combination of a Coulomb plus a square-root kernel. We are able to get good fits to the meson masses with both kinds of kernels and our present results are qualitatively similar to those obtained for the Coulomb plus a linear kernel.

PACS. 12.40.-y Other models for strong interactions

1 Introduction

For some years now, we have been following an approach that combines the concepts and methods of quantum field theory with those of many-body physics in an attempt to investigate the effects of temperature on the dynamics of systems. We have used this approach both in quantum electro-dynamics (QED) and quantum chromo-dynamics (QCD).

In QED, we have considered a system of electrons and protons with the Coulomb interaction between them being modified by temperature [1]. Subsequently, we have applied the results of this study to the solar corona in an attempt to account for the observed wavelengths [2,3] and relative intensities [4] of solar emission lines in the soft X-ray region and also to suggest a possible resolution of the long-standing Rowland puzzle [5].

In QCD, we have attempted to address the problem of the origin of the masses of the meson families. Using the relativistic framework provided by the Bethe-Salpeter (BS) equation with a Coulomb plus a linear kernel, the calculated masses of the Charmonium and Upsilonium families have been shown to be in good agreement with the experimentally measured values, provided the temperature of the quark-gluon plasma is below a threshold temperature of about 100 MeV. Deconfinement in the model was found to occur in the limit of infinite temperatures [6, see also 7].

In the present paper we continue our study of the possible mechanisms that bring about the particlization of the quark-gluon plasma within the relativistic framework spelt out in our earlier work [6]. We now consider interactions between quarks and anti-quarks which reduce in the non-relativistic limit to a logarithmic (Log) or a Coulomb plus a square-root (C-sqr) potential. Since investigations with such interactions have to date only been carried out within a non-relativistic formalism, it seems desirable to examine them within a framework which is not only relativistic but also temperature dependent.

Additional motivation for the study of the Log model is provided by the fact that this model combines the features of a Coulomb-like and a confining interaction in a compact manner. Another attractive feature that this model is likely to possess is that it may be derivable from a field theoretic construct - the gluon superpropagator, which is the propagator for an appropriate non-polynomial function of the gluon field. Work is currently in progress to prove this conjecture.

The procedure for the temperature generalization of the BS equation has been described at length in reference [6] – which we refer to hereafter as paper I and which we ask our readers to consult for details as well as for the notation we use in the present work. To recapitulate briefly the derivation, we assume that successive nucleus-nucleus collisions give rise to a quark-gluon plasma, which may be considered as an ideal fluid in thermal equilibrium at a temperature T. The relativistic nature of the system then suggests that the pair-formation in it be described by a BS equation. To obtain such an equation which also incorporates T, we start with the $T = 0$ BS equation [8]. Taking recourse to the ladder approximation, we go over to the centre-of-mass reference frame of a $q\overline{q}$ pair and carry out the spin reduction of the equation by the method of Gordon [9,10]. We then use the instantaneous approximation which, regardless of the choice of the kernel, enables one to temperature-generalize the equation by following the Matsubara prescription. The equation thus obtained is a three-dimensional integral equation with an as yet unspecified kernel. Then on specializing to the Log and the C-sqr models, in Sects. 2.1 and 2.2 respectively, the corresponding equations are (Fourier) transformed to co-ordinate space and solved numerically to yield the meson masses. In the final section we report our results and discuss their implications.

2 Temperature-dependent dynamics

2.1 Logarithmic kernel

The logarithmic potential as a possible interaction between quarks and anti-quarks was originally proposed by Quigg and Rosner [11] as an alternative to the combination of a Coulomb and a confining potential. However, a detailed analysis of this interaction in a BS framework has not been attempted so far. As pointed out in the Introduction, it is in the choice of the potential describing the $q\overline{q}$ interaction that our present paper differs from paper I. We choose

$$< \mathbf{q}|V_{12}|\mathbf{k} >= \frac{2\pi^2\lambda}{((\mathbf{q}-\mathbf{k})^2)^{3/2}} - \frac{(2\pi)^3 C\delta^3(\mathbf{q}-\mathbf{k})}{m^2}. \quad (1)$$

The first term is the Fourier transform in momentum space of a logarithmic potential and the constant term has been added to simulate a term that arises in the Gordon reduction of the original equation [8].

On using this interaction in (8) of paper I and using the approximation embodied in (14) of that paper we can Fourier transform the resulting equation to obtain, in co-ordinate space, the equation

$$\begin{aligned}
&\left[2m(3-M^2/4m^2)+16\lambda^{'}ln(\rho)/3+16C^{'}/3m^2\right]u^{''}(\rho)\\
&\quad=-(8/3)\Big[(2+M/m)\lambda^{'}/\rho\Big]\,u^{'}(\rho)\\
&\quad+(4\lambda'/3)\Big[(M^2/m_s^2)ln(\rho)\\
&\quad+\big\{3+M/m+4\mathbf{L}\cdot\mathbf{S}-2\mathbf{S}_{12}/3\\
&\quad+4\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2/3+4l(l+1)ln(\rho)\big\}/\rho^2\Big]\,u(\rho)\\
&\quad+(4C^{'}/3m^2)\Big[M^2/m_s^2+4l(l+1)/\rho^2\Big]\,u(\rho)\\
&\quad+4m\Big[(m^2-M^2/4)/m_s^2\\
&\quad+(3/2-M^2/8m^2)l(l+1)/\rho^2\Big]\,u(\rho), \qquad (2)
\end{aligned}$$

where $\rho = m_s r$ is dimensionless, m is the quark mass, $\lambda^{'} = t\lambda$, $C^{'} = tC$ and $t = \tanh(m\beta/2)$. Note that here, as in paper I, the total load of the temperature dependence is carried by the factor $\tanh(m\beta/2)$ multiplying λ and C and that this feature is independent of the potential used for confinement.

2.2 Square-root kernel

We now consider the combination of a Coulomb and a Square-root potential, which so far has only been investigated within the framework of the Schrodinger equation [12]. The choice of the kernel is now

$$\begin{aligned}
< \mathbf{q}|V_{12}|\mathbf{k} > = \frac{4\pi\alpha_s}{(\mathbf{q}-\mathbf{k})^2} + \frac{3\pi^{3/2}\lambda}{\sqrt{2}((\mathbf{q}-\mathbf{k})^2)^{7/4}}\\
-\frac{(2\pi)^3 C\delta^3(\mathbf{q}-\mathbf{k})}{m^2}. \qquad (3)
\end{aligned}$$

Following the same procedure as in the previous subsection, we get the final equation in co-ordinate space in the present case to be

$$\begin{aligned}
&\Big[16\big\{{\alpha_s}^{'}-\lambda^{'}r^{3/2}-C^{'}r/m^2\big\}/3-m(6-M^2/2m^2)r\Big]u^{''}(r)\\
&=\Big[(4/3)(2+M/m)(2\alpha_s^{'}/r+\lambda^{'}r^{1/2})\Big]\,u^{'}(r)\\
&\quad+(4\alpha_s^{'}/3)\Big[M^2+4\pi r\delta^3(\mathbf{r})\,(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+M/m)\\
&\quad-\big\{\mathbf{S}_{12}+4\mathbf{L}\cdot\mathbf{S}+4+2M/m-4l(l+1)\big\}/r^2\Big]\,u(r)\\
&\quad+(4\lambda^{'}/3)\Big[-M^2r^{3/2}-\big\{5/4+M/4m+2\mathbf{L}\cdot\mathbf{S}+\mathbf{S}_{12}/4\\
&\quad+(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)/2+4l(l+1)\big\}r^{-1/2}\Big]u(r)\\
&\quad-\Big[\big\{4C^{'}M^2/3m^2+4m(m^2-M^2/4)\big\}r\\
&\quad+l(l+1)\big\{m(6-M^2/2m^2)+16C^{'}/3m^2\big\}/r\Big]u(r), \quad (4)
\end{aligned}$$

where $\lambda^{'}{=}t\lambda$, $\alpha_s^{'}{=}t\alpha_s$, $C^{'} = tC$ and $t = \tanh(m\beta/2)$.

3 Results and discussion

(2) and (4) are solved numerically using the 4th-order Runge-Kutta-Merson method while ensuring proper boundary conditions. The results of our calculations for the two potentials along with the values of the parameters used are given in Tables 1 and 2. In each case we are able, for the choices of the coupling constants reported, to obtain equally good fits to the experimentally observed masses of the $b\overline{b}$ and $c\overline{c}$ families. That this is so is understandable because a plot of the two potentials for the values of the coupling constants chosen, reveals that even though their analytic forms are very different, the two are practically indistinguishable over a substantial part of the range over which the eigenfunctions belonging to (2) and (4) are non-vanishing. We note that in addition to the masses of the observed mesons we also report the masses of some states which have still not been seen experimentally. These could serve as checks on our models when they are actually observed.

With regard to the temperature dependence of our results, we find that the masses of the bound states are

Table 1. Numerical results for the Logarithmic kernel
$(\Upsilon) : m_b = 4.595$ GeV, $\lambda = 0.487$ GeV, $C = -0.70$ GeV^3
$(\Psi/J) : m_c = 1.510$ GeV, $\lambda = 0.457$ GeV, $C = -1.03$ GeV^3

State $n\left({}^{2S+1}L_J\right)$	M_{exp} (GeV) (Υ)	M_{theo} (GeV) (Υ)	M_{exp} (GeV) (Ψ/J)	M_{theo} (GeV) (Ψ/J)
1^3S_1	9.460	9.467	3.097	3.269
2^3S_1	10.023	10.011	3.686	3.754
3^3S_1	10.355	10.361	4.040	4.123
4^3S_1	10.580	10.626	4.415	4.434
5^3S_1	10.865	10.843	.....	4.704
6^3S_1	11.019	11.027	.....	4.959
1^1S_0	9.377	9.279	2.979	2.954
2^1S_0	9.963	9.905	.....	3.611
3^1S_0	10.298	10.282	.....	4.040
4^1S_0	10.573	10.561	.....	4.380
1^3P_0	9.859	9.841	3.415	3.438
2^3P_0	10.232	10.233	.....	3.796
1^3P_1	9.891	9.841	3.510	3.438
2^3P_1	10.255	10.233	.....	3.796
1^3P_2	9.913	9.884	3.556	3.620
2^3P_2	10.268	10.264	.....	3.960
1^3D_1	......	10.139	3.770	3.730
2^3D_2	......	10.444	4.159	3.954

independent of temperature below about 100 MeV and that in the high temperature limit both models show deconfinement – features that were reported in Paper I in the case of the Coulomb plus a linear potential. In fact, we have verified that the detailed dependence of the masses of the bound states on temperature for all three models are qualitatively similar. Another feature of our calculations is that they take explicit account of spin, which comes about because we use the method of Gordon in the spin-reduction of our equations.

The results obtained here confirm that the logarithmic potential as proposed by Quigg and Rosner [11] and the combination of a Coulomb with a square-root potential proposed by De Carvalho [12] are viable alternatives to the Cornell potential in that they lead to equally adequate explanations of the masses of the Charmonium and Upsilonium families of mesons. However, the fact that different sets of values of the coupling constants are required to fit the masses of the different families of mesons for each of the potentials indicates that the strength of the interaction between the quark anti-quark pairs are not flavour independent. Our preliminary attempts to fit the masses of the $u\overline{u}$ and $s\overline{s}$ states using these potentials further strengthens this conclusion.

Table 2. Numerical results for the Coulomb plus a Square-root kernel
$(\Upsilon) : m_b = 4.540$ GeV, $\alpha_s = 0.115$, $\lambda = 0.500$ GeV$^{3/2}$, $C = -6.715$ GeV^3
$(\Psi/J) : m_c = 1.540$ GeV, $\alpha_s = 0.115$, $\lambda = 0.500$ GeV$^{3/2}$, $C = -1.750$ GeV^3

State $n\left({}^{2S+1}L_J\right)$	M_{exp} (GeV) (Υ)	M_{theo} (GeV) (Υ)	M_{exp} (GeV) (Ψ/J)	M_{theo} (GeV) (Ψ/J)
1^3S_1	9.460	9.489	3.097	3.215
2^3S_1	10.023	9.971	3.686	3.672
3^3S_1	10.355	10.316	4.040	4.029
4^3S_1	10.580	10.602	4.415	4.329
5^3S_1	10.865	10.852	.....	4.602
6^3S_1	11.019	11.077	.....	4.888
1^1S_0	9.377	9.464	2.979	3.138
2^1S_0	9.963	9.957	.....	3.623
3^1S_0	10.298	10.305	.....	3.993
4^1S_0	10.573	10.593	.....	4.302
1^3P_0	9.859	9.789	3.415	3.244
2^3P_0	10.232	10.159	.....	3.607
1^3P_1	9.891	9.817	3.510	3.452
2^3P_1	10.255	10.186	.....	3.793
1^3P_2	9.913	9.837	3.556	3.536
2^3P_2	10.268	10.202	.....	3.881
1^3D_1	......	10.065	3.770	3.662
2^3D_2	......	10.379	4.159	3.879

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